Steel Interchange

Design of Cantilever Beams

The following is a brief paper presenting one engineer's opinion on design parameters for use with cantilever beams. The author has included a list of pertinent reference sources on the subject.
Cantilever Flexural Member Design
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Answer

Introduction>
The AISC 1999 Load and Resistance Factor Design Specification for Steel Buildings(1) has no specific flexural design requirements for cantilever beams beyond requiring Cb = 1 when the free end is unbraced. A review of the literature on cantilever analysis reveals this minimal requirement may not be enough to steer the engineer from creating cantilever designs that, while meeting the letter of the specification, at times may be unconservative.
Nethercot(2,3) has done extensive research on cantilever analysis and design. His relevant findings are summarized in the Guide to Stability Design Criteria for Metal Structures(4). Nethercot's approach is to use an effective length factor, designated Kc, to account for a variety of restraint and loading arrangements. Kc values can range from less than one to greater than one.

Based on Nethercot's work, one can modify the 1999 LRFD Specification equations as follows for use with cantilevered wide-flange beams:

$$L_r = \frac{r_y X_1}{K_c F_L}\sqrt{1+\sqrt{1+X_2 F_L^2}}$$

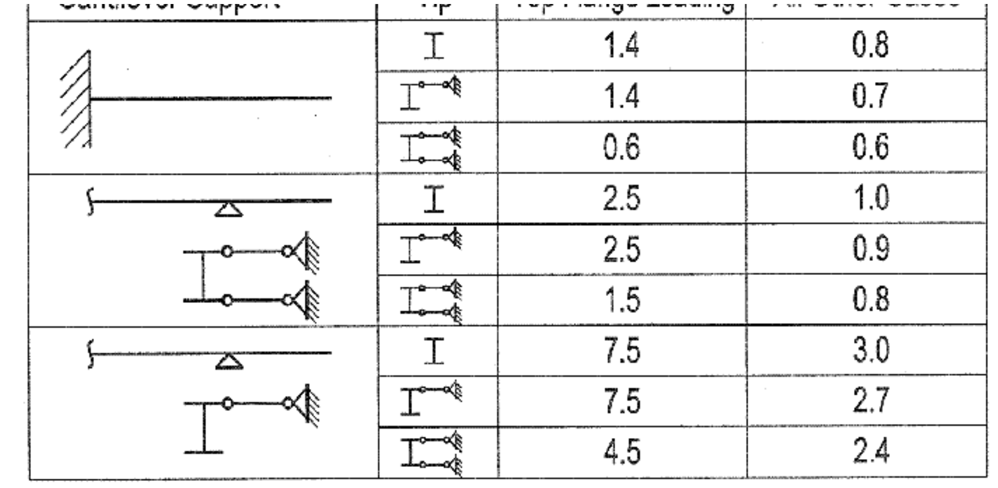
Equation F1-6:

$$M_{cr} = C_b \frac{\pi}{K_c L_b}\sqrt{E I_y G J + \left(\frac{\pi E}{K_c L_b}\right)^2 I_y C_w}$$

Equation F1-13:

Figure 1
K_c Values for Cantilevers

Restraint Continuous		K_c	
Cantilever Support	Tip	Top Flange Loading	All Other Cases



1.4	0.8
1.4	0.7
0.6	0.6
2.5	1.0
2.5	0.9
1.5	0.8
7.5	3.0
7.5	2.7
4.5	2.4

where Kc is given by Figure 1 (above, adapted from Nethercot); and Cb = 1 for any cantilever section, regardless of bracing conditions.

Bracing Type and Location

Traditionally, engineers expect that bracing should be of the compression flange. However, in the case of the cantilever, it is the tension flange that deforms most during bucking. Bracing a cantilever beam's compression flange alone has almost no impact on the beam's stability.

Kitipornchai(5) studied efficiency of bracing types and location along the span. He found that for discrete lateral bracing the most effective location is the top (tension) flange and as close to the cantilever tip as practical. He also found that for cantilevers fully restrained at the support, lateral restraints placed within the first 40% of the span are practically useless. Until more research is done on multiple or continuous restraints along the cantilever length, it seems prudent to consider only the restraint provided at the support and the tip.

Kc and Cb

Figure 1, a modified form of tables given by Nethercot, identifies Kc values that were derived assuming Cb = 1. Nethercot has published work in which Kc = 1 and the Cb factor is used to adjust for restraint and loading conditions at the cantilever, but using an effective length factor seems more intuitive and Cb = 1 should be used when using the Kc values from Figure 1. (2,3,6)

Conclusion

Let's consider one case where this approach is relevant. An under-slung crane beam is projecting out from the side of a building. Inside the building, the beam is over a mechanical space and is supported by roof beams. The tip of the crane beam extends outside the building such that deliveries can be hoisted up and brought into the building. The hoisting device has wheels that run on the bottom flange of the beam, so no stiffeners or bracing can be provided to the bottom flange of the cantilever beam without interfering with the operation of the crane device. Further, the architect is adamant that you are not to provide braces to the flanges of the cantilever beam tip at the exterior of the building. Not only does he feel this will not look good, but he's also driven around and seen this condition without them. You don't want him going to another engineer because he might not come back.

In this case, you have loading that is not top flange loading, the tip is unbraced at the top and bottom, and the "root" of the cantilever beam is only braced at the top flange. A Kc value of 3.0 is selected, and the beam is designed using the formulas above.

There are several other methods out there(8,9). Some of them are more exact. However, in each case the methods are situation specific, are a bit too complex to be used in an office setting, or just do not cover enough cases. The nice thing about Nethercot's work is that in a straightforward way it covers most possible restraint conditions and loading conditions, and it is conservative. (Note that built into the Kc values are the effects of skip loading, uniform loading, point loading, varying ratios of back span length to cantilever length, varying support conditions for the "far" end of the supported portion of the beam, etc. He just used the worst case).

As a final comment, the proposed method lends itself to expansion to cover other problematic areas. Tables similar to Figure 1 could be developed to aid in the design of continuous beams and laterally unsupported beams with varying end restraints.

Acknowledgments

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Cited References

1. AISC (1999), Load and Resistance Factor Design for Structural Steel Buildings, AISC, Chicago.
2. Nethercot, D.A. (1973), "The Effective Length of Cantilevers as Governed by Lateral Buckling," The Structural Engineer, Vol. 51, No. 5.
3. Nethercot, D.A. (1983), "Elastic Lateral Buckling of Beams," Beams and Beam-Columns: Stability and Strength (ed. R. Narayanan), Applied Science Publishers, Barking, Essex, England.
4. Galambos, T.V (ed.) (1998), Guide to Stability Design Criteria for Metal Structures, Structural Stability Research Council, 5th Edition, John Wiley and Sons, New York.
5. Kitipornchai, Dux, Richter (1984), "Buckling and Bracing of Cantilevers," Journal of Structural Engineering, Vol. 110, No.9
6. Salmon and Johnson (1996) Steel Structures -- Design and Behavior, 3rd Edition, p. 511, HarperCollins, New York.
7. AISC (1999), Manual of Steel Construction, Load and Resistance Factor Design, Third Edition, AISC, Chicago.
8. Essa and Kennedy (1995), "Design of Steel Beams in Cantilever-Suspended-Span Construction," Journal of Structural Engineering, Vol. 121, No.11
9. Bo Dowswell (2004) "Lateral-Torsional Buckling of Wide Flange Cantilever Beams," Engineering Journal, Vol. 41, No. 2.

Additional References

- Hemstad, Michael (1999), "Cantilever Beam Framing Systems," Engineering Journal, Third Quarter.
- Nethercot, D.A. and Kirby, P.A. (1979), Design for Structural Stability, John Wiley and Sons, New York.
- Timoshenko and Gere (1961), Theory of Elastic Stability, McGraw-Hill, New York. Yura, Joseph (1995), "Bracing for Stability State-of-the-Art," Proceedings, Structural Congress XIII, ASCE, Boston.
- Yura, Joseph (2001), "Fundamentals of Beam Bracing," Engineering Journal, First Quarter.

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